Adapting to Climate Change

by

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Abstract

This paper examines the optimal time to adapt to climate change. We take the perspective of a farmer growing a crop in a stochastic environment. The farmer faces stochastic seasonal growth, which makes harvest at the end of any season a random variable. Within a season, crop biomass is assumed to grow according to a continuous-time Itô process. The standard deviation rate of the Itô process is itself stochastically evolving, season to season, as a result of climate change. We assume the seasonal standard deviation rate follows a discrete-time random walk, with positive drift. As the seasonal standard deviation rate grows, expected biomass at harvest, and thus revenue, declines. The farmer has the option to make an investment, say in an irrigation system, which will reduce the seasonal standard deviation rate. The investment in irrigation has a fixed cost and also results in higher cultivation costs during a season. The question becomes "How large must seasonal variation become before it is optimal to make the investment and adapt to climate change?"

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Adapting to Climate Change

Potential changes in the frequency, intensity, and persistence of climate extremes (for example, heat waves, heavy precipitation, and drought) and in climate variability, … are emerging as key determinants of future impacts and vulnerability.

Intergovernmental Panel on Climate Change (2001)

Introduction and Overview

By the year 2100, many of the scientists studying climate change believe that the increased concentration of greenhouse gases will lead to an increase in annual global mean surface temperature in the range of 1.4 to 5.8º C. Theory and mathematical models also project an increase in climate variability, specifically the frequency of "climate extremes." It is the increase in climate variability that raises the greatest potential for adverse impacts within human (socioeconomic) and ecological systems. The vulnerability of different human populations and plant and animal species will depend on the speed of climate change and on the ability to adapt.

Adaptation to climate change can take many forms. In agriculture, it may involve the adoption of later maturing cultivars, changing the mix of crops, or altering the timing of field operations [Kaiser *et al.* (1993)] . In the extreme, it may involve abandonment of land and human migration, as in the dust bowl of the U.S. Midwest during the 1930s or in Africa today.

This paper is concerned with the timing of capital investments undertaken as a means of adapting to climate change. We assume that such investments will involve an initial capital cost and perhaps higher variable cost during cultivation after investment. It is further assumed that the investment may be so specialized that, if not technically irreversible, it would be costly to reverse, and any scrap value would be a small fraction of the initial capital cost. Two investments come to mind. Irrigation equipment, adopted because of the increased frequency or duration of drought, might be removed and sold at a later date. Investment in better drainage, for a field now subject to higher levels of precipitation, might be costly to undo, and the drainage tile itself is likely to be worthless as scrap.

We assume these investments are risky, in the sense that their use and value will still vary from season to season, and thus the future net return from making the investment is not known with certainty. A farmer, contemplating an investment to cope with climate change, is faced with the classic economic problem of risky, irreversible investment [Dixit and Pindyck (1994)]. The proper evaluation of such investments requires their analysis as real options [Trigeorgis (1996)].

This paper is organized as follows. In the next section we develop two models. The first is an infinite-horizon model, which might be appropriate for a corporate or family farm with the expectation of long-term operation. The second model is a finite-horizon model, which might be appropriate for a farmer with no heirs interested in continued farming, and with a plan for selling land and equipment at some future date as a source of retirement income. This section is followed by numerical analysis for a hypothetical farm growing a single crop. In the infinite-horizon model there is a single standard deviation rate that triggers the corporate or family farm to make the investment to adapt to a more variable climate. In the finite-horizon problem, with the farmer planning divestment prior to retirement, there is a schedule of critical standard deviation rates which increase as the farmer nears retirement. Quite logically, the farmer is less interested in a costly investment to cope with a more variable climate the nearer he or she is to retirement. The final section gathers conclusions and suggests future lines of research.

The Infinite- and Finite-Horizon Models

Let X=X(t) denote the biomass of a crop during a growing season. We assume a continuous, intra-season, stochastic growth process given by

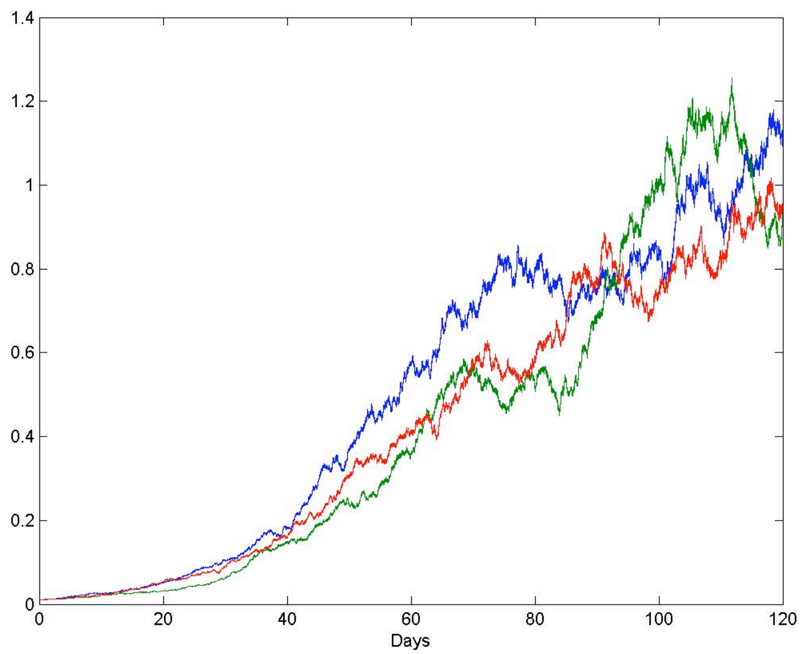
$$dX = rX(1 - X/K)dt + \sigma_s X dz \tag{1}$$

where $r>0$ is an intrinsic growth rate, $K>0$ is the maximum crop biomass at the end of a season if there were no stochasticity to growth, σ_s is the standard deviation rate during season s, and dz is the increment of a Wiener process. Solving the Kolmogorov forward equation for the steady state density of crop biomass, Merton (1975) and Dixit and Pindyck (1994) show that X(t) will have a gamma distribution as $t \to \infty$. The expected crop biomass, at the end of season s, may be approximated by

$$E\{X_s\} = K(1 - \sigma_s^2/(2r)) \tag{2}$$

By proper selection of the time step and appropriately scaling of the intrinsic growth rate, r, Equation (2) will provide a good approximation for expected harvest during season s, when the standard deviation rate is σ_s. Note, if σ_s increases from season to season, expected harvest declines. In Figure 1 we show three realizations, from a sample of 10,000 realizations, where $X_0 = 0.01$, $\Delta t=0.001$, $r=0.08$, $K=1$, $\sigma_s=0.05$ and the growing season was $T=120$ days. The mean harvest, for all 10,000 realizations, was 0.8935, with a standard deviation of 0.1369; not significantly different from $K(1-\sigma_s^2/2r)=0.9844$.

Figure 1. Three Realizations of Crop Biomass



Suppose, because of climate change, that the standard deviation rate is evolving according to a discrete random walk with drift as given by

$$\sigma_{s+1} = \mu + \sigma_s + \varepsilon_{s+1} \tag{3}$$

where $\mu>0$ is the drift rate in the seasonal standard deviation of crop biomass and $\varepsilon_{s+1} \sim N(0,\sigma_\varepsilon^2)$. It can be shown that the distribution of σ_{s+1}, conditional on σ_s, is given by

$$f(\sigma_{s+1} \mid \sigma_s) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-(\sigma_{s+1}-(\mu+\sigma_s))^2/2\sigma_\varepsilon^2} \tag{4}$$

At the start of season s, with r, K, and σ_s known, the farmer has an expected net revenue of

$$N_s = pK(1-\sigma_s^2/(2r)) - c \tag{5}$$

where $p>0$ is the per unit commodity price at harvest and $c>0$ is the cost of cultivation and harvest.

Suppose the farm is owned by a corporation or a family with an expectation of multi-generational succession. Suppose further that the corporation or family can make an investment which will stabilize (fix) the seasonal standard deviation in crop growth at $\sigma_I < \sqrt{2r}$. If the investment is adopted after season s and is in place for the start of season $s+1$, the *expected discounted net revenue with the investment* would be given by

$$N = N_s - I + \sum_{s=1}^{\infty}\rho^s[pK(1-\sigma_I^2/(2r)) - k] = N_s - I + [pK(1-\sigma_I^2/(2r)) - k]/\delta \tag{6}$$

where $\rho=1/(1+\delta)$ is the annual discount factor, $\delta>0$ is the annual discount rate, and $k>c$ is the cultivation and harvest cost *with* the investment. The *value function* for a farmer with the option to invest depends on the current standard deviation rate and is given by

$$V(\sigma_s) = \text{Max}\{N_s + \rho E[V(\sigma_{s+1} \mid \sigma_s)],\ N\} \tag{7}$$

where the term $\rho E[V(\sigma_{t+1} \mid \sigma_s)]$ is the expected discounted value of *not* investing at the end of season s but preserving the option to invest at the end of season $s+1$. In this infinite-horizon problem there will exist a critical standard deviation, σ^*, which makes it optimal for the corporation or family to pay the fixed cost I and incur the higher cultivation cost k in order to take advantage of the lower standard deviation rate, $\sigma_I < \sigma^*$. It is not possible to derive an analytic expression for σ^*, but we will be able to solve for it numerically as part of the solution to the finite-horizon model, which we present next.

Now consider the problem for a farmer who is age 30 in season $s=0$, and who plans to retire at the end of season $S=35$, at which time he will sell the land for L dollars and receive S dollars in scrap value if an investment was made in any season $0 \leq s < S$. In the finite-horizon problem we need to numerically solve for the value function $V(s,\sigma_s)$. Since the last season that the investment could be made is $s=34$, we start by considering the boundary values for $V(34,\sigma_{34})$ assuming that the investment has *not* been adopted prior to $s=34$ and that $0 \leq \sigma_{34} \leq \sqrt{2r}$. Then,

$$V(34,\sigma_{34}) = \text{Max}[N_{34} + \rho[E[V(35,\sigma_{35} \mid \sigma_{34})] + L],\ N_{34} - I + \rho[pK(1-\sigma_I^2/(2r)) - k + L + S]] \tag{8}$$

The other boundaries are V(s,0) and $V(s,\sqrt{2r})$ for s=0,1,…,33. When σ_s=0,

$$V(s,0) = pK - c + \rho E[V(s+1,\sigma_{s+1} \mid 0)] \tag{9}$$

When $\sigma_s = \sqrt{2r}$ expected harvest is zero in season s ($E\{X_s\}$=0) and we assume there is no cultivation nor harvest cost in season s. The farmer is faced with a decision of making the investment to reduce the standard deviation to $\sigma_I < \sqrt{2r}$, or to leave the land fallow in season s and hope that the standard deviation rate stochastically declines in season s+1. This yields the boundary condition

$$V(s,\sqrt{2r}) = \text{Max}[\rho E[V(s+1,\sigma_{s+1} \mid \sigma_s)], -I + [pK(1-\sigma_I^2/(2r) - k](1-\rho^{35-s})/\delta + \rho^{35-s}(L+S)] \tag{10}$$

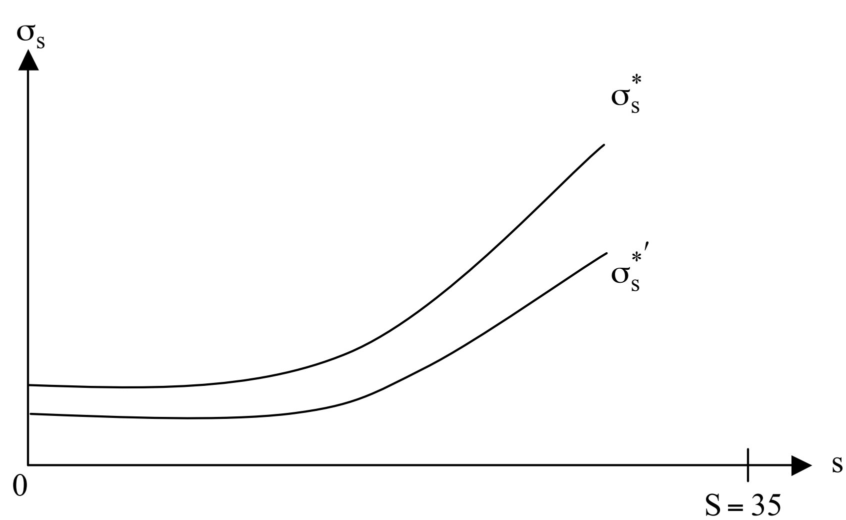
With boundary conditions (8) – (10) we can solve for the "interior" values V(s,σ_s) using stochastic dynamic programming. These values will satisfy the H-J-B equation

$$\begin{aligned} V(s,\sigma_s) = \text{Max}[&N_s + \rho E\{V(s+1,\sigma_{s+1} \mid \sigma_s)\}, \\ &N_s - I + [pK(1-\sigma_I^2/(2r) - k](1-\rho^{35-s})/\delta + \rho^{35-s}(L+S),] \end{aligned} \tag{11}$$

We expect that the critical standard deviation rate, σ_s^*, will increase with age (as the farmer nears retirement). The intuition is obvious. Even through a more variable climate may be reducing expected crop yield, and thus net revenue, σ_s would have to be higher as the farmer nears retirement to induce him to pay the fixed cost, $I>0$, when there may be only a few seasons left, at σ_I, to recoup that fixed cost via higher yields and larger net revenues.

We also have economic intuition about the effect that a change in μ, the rate of climate change, I, the cost of investment to adapt to climate change, k, the variable cost of operating the new investment, S, the scrap value of the investment upon retirement, and σ_I, the post-investment standard deviation rate. If μ or S increase, or if I, k or σ_I decrease, the entire σ_s^* -schedule will shift downward in $(s\text{-}\sigma_s)$ space. See $\sigma_s^{*\prime}$ in Figure 2 below.

Figure 2. The critical σ_s^*-Schedule for a Finite-Horizon Problem



Numerical Analysis of a Hypothetical Crop

In this section, we present some numerical results for a hypothetical crop and a variance-reducing investment. We solve the finite-horizon problem for the σ_s^* schedule. For a young farmer, say age 30 in season s=0, the value σ_0^* may give us an approximation of the unique critical standard deviation rate, σ^*, for the infinite-horizon problem. We return to this aspect at the end of this section.

We assume crop biomass is an Itô variable whose evolution is defined by Equation (1) with $r=0.08$ and $K=1$. The seasonal standard deviation rate, σ_s, evolves according to the random walk with drift as given by Equation (3). We assume that $\mu=0.0025$ and the random variates, ε_{s+1} are independently drawn from a normal distribution with mean zero and standard deviation $\sigma_\varepsilon=0.01$.

Equation (3) is approximated by a Markov transition matrix. We partition the interval $\sigma_s \in \left[0, \sqrt{2r}\right]$ into M equally-spaced points. Let index i be $i \in \left\{0,1\ldots,M\right\}$, and let $\sigma_i = i \cdot \Delta\sigma$ for $\Delta\sigma = \sqrt{2r}/M$. All possible values for the standard deviation rate, σ_s, are represented by the $M+1$ points σ_i, $i = 0,1\ldots,M$. The Markov transition matrix is an $(M+1)\times(M+1)$ matrix with elements P_{ij}, for $i, j = 0,1\ldots,M$, where P_{ij} represents the probability that $\sigma_{s+1}=\sigma_j$ given that $\sigma_s=\sigma_i$. The transition probabilities can be easily computed from Equation (4).

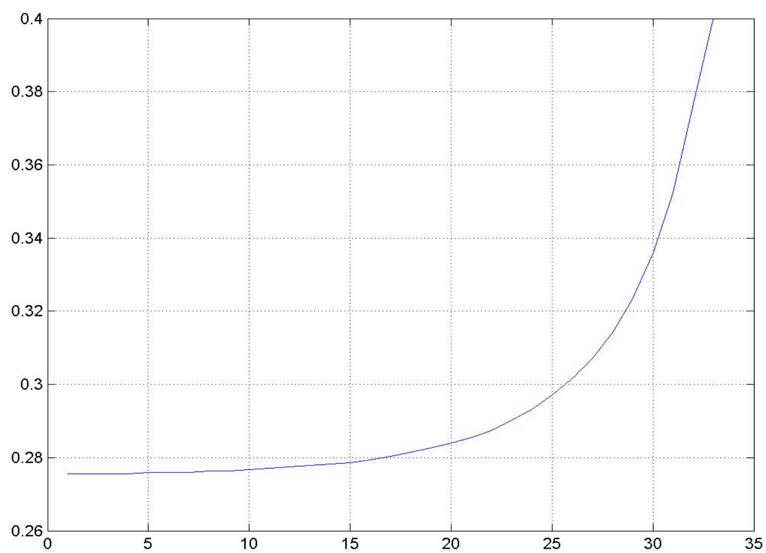
Given the Markov transition matrix, the H-J-B equation is reformulated as

$$V(s,\sigma_i) = \text{Max}[N_s + \rho \sum_{j=0}^{M} P_{ij} V(s+1, \sigma_j \mid \sigma_i),$$
$$N_s - I + [pK(1-\sigma_I^2/(2r)) - k](1-\rho^{35-s})/\delta + \rho^{35-s}(L+S)] \qquad (12)$$

As $M \to \infty$, the solution to Equation (12) converges to that of Equation (11). Using Equation (12), the numerical values for $V(s,\sigma_s)$ are obtained by solving backward from $s=34$ to $s=0$. The values, σ_s^*, which trigger investment for each season (age=30+s) are recorded and curves, similar to those posited in Figure 2 can be drawn.

For the base-case, in addition to $r=0.08$, $K=1$, $\mu=0.0025$ and $\sigma_\varepsilon=0.01$, we set $I=30$, $k=4$, $c=2$, $L=50$, $S=10$, $\rho=0.95$, $\sigma_I=0.12$, and $p=10$. The base-case σ_s^* -schedule is shown in Figure 3.

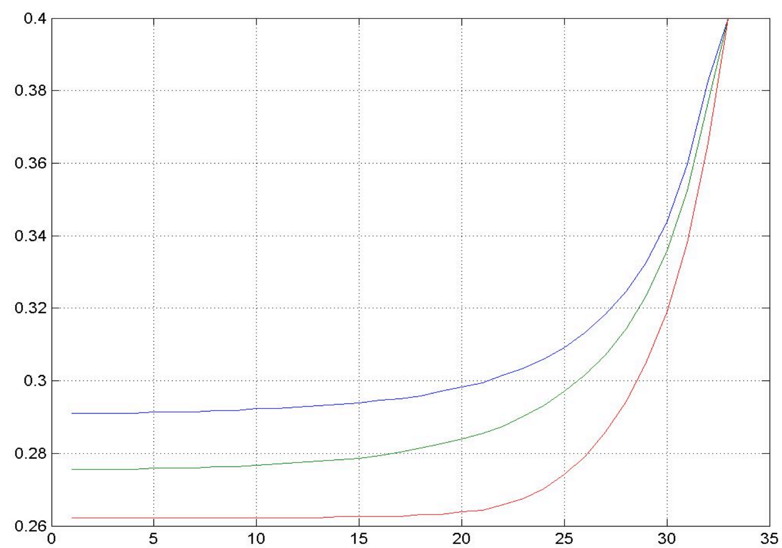
Figure 3. The Base-Case σ_s^* - Schedule



Next, we do some sensitivity analysis, to verify our economic intuition, by changing a single parameter in the base-case parameter set. For μ, I, and k, we show the change in the σ_s^* - schedule for values above and below their base-case setting.

For μ, the drift rate for seasonal standard deviation, we compute the σ_s^* - schedule for $\mu = \{0.00, 0.0025, 0.005\}$. All other base-case parameters are unchanged. Figure 4 shows the results, where blue, green and red lines depict the σ_s^* - schedules for $\mu = \{0.00, 0.0025, 0.005\}$, respectively. Note, as μ increases, the σ_s^* - schedule shifts downward.

Figure 4. The σ_s^* - Schedules for $\mu = \{0.00, 0.0025, 0.005\}$



Next we compute the σ_s^* - schedules for $I = \{15, 30, 45\}$. Figure 5 shows the results, where blue, green and red lines show the σ_s^*-schedules for $I = \{15, 30, 45\}$, respectively. Note, as I increases, the σ_s^* - schedule shifts upward.

Finally, we changed the value of k to analyze how the difference between the cultivation cost *with* the investment and the cultivation cost *without* the investment $(k-c)>0$, affects the σ_s^* -schedule. The three values were $k=\{2,4,6\}$. Figure 6 shows the results, where blue, green and red lines show the σ_s^*-schedules for $k=\{2,4,6\}$, respectively. Note, larger values of k cause the σ_s^* - schedule to shift upward and that the σ_s^* -schedule is especially sensitive (for our hypothetical crop) to increases in k.

Figure 5. The σ_s^* - Schedules for $I=\{15,30,45\}$

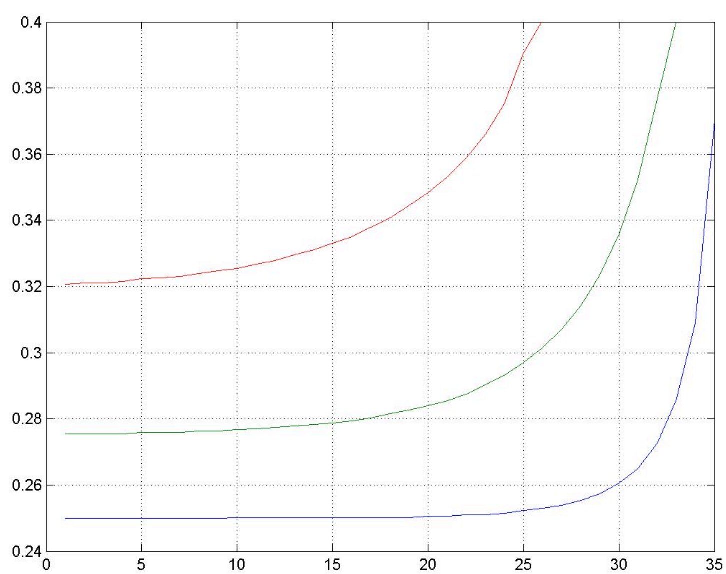
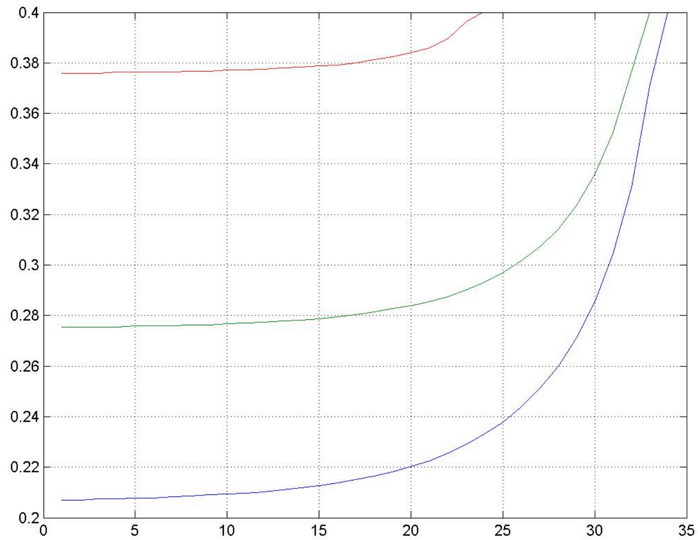


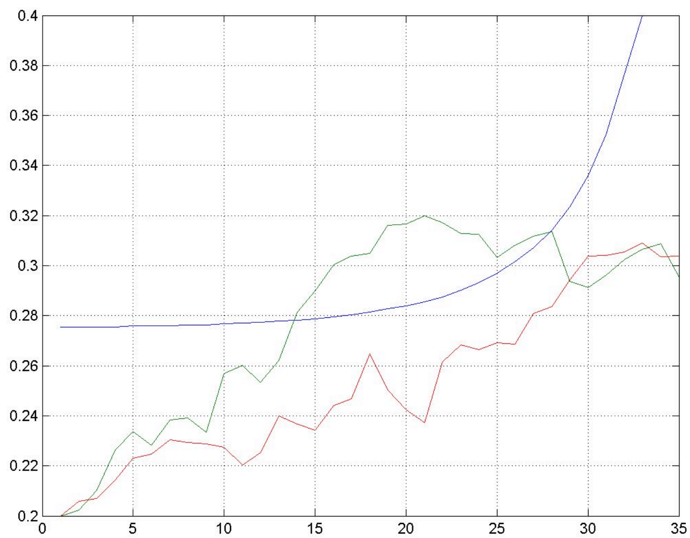
Figure 6. The σ_s^* - Schedules for $k = \{2,4,6\}$



We do not show the comparative statics for σ_I and S, the seasonal standard deviation rate after making the investment, and the scrap value of the investment upon retirement. We simply note that as σ_I decreases or S increases, other things equal, the σ_s^* - schedule numerically shifts downward, as economic intuition would suggest.

Given a σ_s^* -schedule, it is important to keep in mind that the farmer does not invest unless a σ_s- realization "hits" or exceeds the critical value for season $s = 1,2,\ldots S$, assuming $\sigma_0 = 0.2 < \sigma_0^*$. In Figure 7, two sample realizations were generated using the Markov Matrix approximation for Equation (3). The σ_s^* - schedule, in blue, corresponds to our base-case set of parameters. The green σ_s-realization hits the σ_s^* -schedule at s=13, while the red σ_s -realization never reaches the σ_s^* -schedule and the farmer never finds it optimal to adopt the investment. So, while we can numerically determine the optimal σ_s^* - schedule for any given parameter set, whether our hypothetical farmer invests or not depends on the particular realization, or "climatic sample" the farmer experiences.

Figure 7. Invest If and When $\sigma_s \geq \sigma_s^*$: Base-Case Parameters and $\sigma_0 = 0.2$



Finally, we note that if the σ_s^* - schedule is horizontal for the early seasons (of a young farmer) that σ_0^* will approximately equal the unique trigger value, σ*, for the infinite-horizon problem. This will be the case if the discounted value of the investment over a 35-year horizon approximates the discounted value over an infinite horizon. In Table 1 we report comparative statics for the infinite-horizon problem. Here we choose a sufficiently long period, $s = 100$, to approximate the unique value σ*. The σ_s^* - schedule is horizontal as s approaches zero (the first season). The same value for σ_0^* is also obtained when we further lengthen the finite horizon to s=500.

Table 1. Comparative Statics for σ^* in the Infinite-Horizon Problem

Parameter	r	K	μ	σ_ε	p	c	σ_I	δ	k	I
σ^*	+	-	-	+	-	-	+	+	+	+

For the infinite-horizon problem, an increase in K, μ, p, or c, will reduce σ^*, while an increase in r, σ_ε, σ_I, δ, k, or I will increase σ^*.

Conclusions and Future Research

In this paper we have developed a model that answers the question "When should a farmer make a costly investment in response to climate change?" We feel the model has many features that make it realistic and a "vehicle" for applied/empirical research. First, we modeled crop growth within a season as a continuous stochastic process. The standard deviation rate, σ_s, has the plausible effect of reducing expected harvest at the end of a growing season. The model assumed that the seasonal standard deviation rate for crop growth was itself a random variable, evolving, season to season, according to a discrete-time random walk. If the growing atmospheric concentration of CO_2 is causing the climate to become more variable, we would presume that the discrete-time random walk for σ_s has a positive drift ($\mu>0$). The climate-adapting investment was viewed as a way to reduce seasonal variance, increase expected harvest, and thus revenue. For an infinite-horizon problem, suitable for a corporate or long-lived family farm, there will be a single critical standard deviation rate that triggers investment. For a single farmer, with no agricultural heirs, there will be a σ_s^* - schedule, with the critical standard deviation rate, that triggers investment, increasing as the farmer nears retirement.

MATLAB programs where written to numerically solve the finite-horizon problem for a hypothetical crop and farmer. Sensitivity analysis was conducted to confirm that the model, specifically shifts in the σ_s^* - schedule, was consistent with economic intuition.

The task now is to find an appropriate panel data set which would allow the estimation of model parameters. The ideal data set would contain crop biomass measures for a randomized set of non-irrigated and irrigated plots, over time. Such a panel would allow estimation of r, K, and the standard deviation rates for crop biomass, within a season, for both non-irrigated (σ_s) and irrigated (σ_I) plots. Time-series estimates of the standard deviation rate for non-irrigated plots would, in turn, permit the estimation of μ and σ_ε. It would also be possible to test whether σ_I is stationary over time. We will be surveying crop scientists to see if such a panel data set exists, and if not, how the model might be calibrated based on a statistical analysis of the available data.

References

Dixit, A., and R. S. Pindyck. 1994. *Investment Under Uncertainty*, Princeton University Press, Princeton, New Jersey.

Intergovernmental Panel on Climate Change. 2001. *Climate Change 2001: Impacts, Adaptation, and Vulnerability*, Cambridge University Press, Cambridge, United Kingdom.

Kaiser, H. M., Riha, S. J., Wilks, D. S., Rossiter, D. G., and R. Sampath. 1993. "A Farm-Level Analysis of Economic and Agronomic Impacts of Gradual Climate Warming," *American Journal of Agricultural Economics*, 75(May):387-398.

Merton, R. C. 1975. "An Asymptotic Theory of Growth Under Uncertainty," *Review of Economic Studies*, 42(3):375-393.

Trigeorgis, L. 1996. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, MIT Press, Cambridge, Massachusetts.