Parallel Performance
Parallel Speedup

- Parallel speedup: how much faster does my code run in parallel compared to serial?
- Max value is $p$, the number of processors

\[
\text{Parallel speedup} = \frac{\text{serial execution time}}{\text{parallel execution time}}
\]
Parallel Efficiency

- Parallel efficiency: how much faster does my code run in parallel compared to *linear speedup from serial*?
- Max value is 1

\[
\text{Parallel efficiency} = \frac{\text{Parallel speedup}}{p}
\]
Model of Parallel Execution Time

- Let $n$ represent the problem size and $p$ be the number of processors
- Execution time of the inherently sequential portion of the code is $\sigma(n)$
- Execution time of the parallelizable portion of the code $\phi(n)/p$
- Overhead is due parallel communication, synchronization is $\kappa(n,p)$

\[ T(n,p) = \sigma(n) + \phi(n)/p + \kappa(n,p) \]
Model of Parallel Speedup

- Divide $T(n,1)$ by $T(n,p)$
- Cute notation by Quinn: $\psi$ (or Greek psi) = parallel speedup

$$\psi(n,p) = \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n) / p + \kappa(n,p)}$$
Amdahl’s Law

- Parallel overhead $\kappa$ is always positive, so...
- Can express it in terms of sequential fraction $f = \sigma(n) / [\sigma(n) + \varphi(n)]$
- Divide numerator, denominator by $[\sigma(n) + \varphi(n)]$

$$\psi(n,p) < \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n) / p}$$
Other Expressions for Amdahl’s Law

\[ \psi(n,p) < \frac{1}{f + (1 - f) / p} \]

\[ \psi(n,p) < \frac{p}{1 + f(p - 1)} \]
Parallel Overhead: Synchronization (Example)

- Suppose the parallelizable part of the code consists of \( n \) tasks each taking the same time \( t \)
- What if \( n \) is not divisible by \( p \)? Let \( x = \text{mod}(n,p) \) = the “extra” tasks
- Sequential execution time: \( t \, n = t \, (n - x) + t \, x \)
- For parallel execution, only the first term shrinks inversely with \( p \)
- For \( p \) processors, the second term takes either 0 or \( t \) (because \( x < p \))
- Therefore, time on \( p \) processors = \( t \, (n - x) / p + t \, \{1 - \delta_{0,x}\} \)

\[
T(n,p) = \sigma(n) + \varphi(n) / p + \kappa(n,p)
\]

\[
\varphi(n) = t \, [n - \text{mod}(n,p)], \quad \kappa(n,p) = t \, \{1 - \delta_{0,\text{mod}(n,p)}\}
\]
Aside: How Do You Code a Kronecker Delta in C?

- Simple once you see it… not so simple to come up with it…
- Formula assumes $0 \leq x < p$

$$\text{Kron}(0,x) = 1 - \frac{(x + (p-x)\%p)}{p}$$
Parallel Overhead: Jitter (Example)
Communication

• Latency
• Bandwidth