Solar-Terrestrial Physics –

The Sun’s Atmosphere, Solar Wind, and the Sun-Earth Connection
The Solar Corona is the Sun’s Extended Atmosphere
Scattered light makes it visible during a total eclipse of the Sun

“helmet streamers”
X-Rays Reveal 3D Magnetic Loops and Arches
The corona is full of magnetic structures at all scales

http://www.lmsal.com/SXT/homepage.html
Close-Up of Some Magnetic Loops

Data from the TRACE satellite at 171 Å (EUV)

*QuickTime movie of Yohkoh SXT images shows the 3D structure of magnetic loops
Coronal Holes

Usually found at the poles, they can extend to lower latitudes

Source: Yohkoh Science Team
The Corona is a Very Dynamic Place!
The Restless Corona (from SOHO)
Solar Flares
Plasma catastrophes trigger bursts of radiation
Flares Often Occur Along Coronal Arcades

An arcade marks a seam between regions of opposite polarity. Shear motion along the seam can cause it to flare all at once.

(TRACE image)
Flare Movie from SDO
First one flash, then more, then a shock that rearranges the global field!

*See the sequence in the QuickTime movies
Flares Are Also Associated with Flux Emergence

The Corona in X-Rays from Solar Max to Min

Sunspots and Active Regions

This was the most highly resolved solar image ever taken by the 1-meter Swedish Solar Telescope (SST) on La Palma.

- Dark patches: umbrae
- Less-dark streaks: penumbrae

Credit: Royal Swedish Academy of Sciences, 2002
X-Ray Emission Above a Sunspot
The chromosphere lies just above the photosphere. Here, magnetic features are highlighted by spectral lines like H\textalpha, Ca II K, and Ca II H (image at right). When viewed in H\textalpha, bright areas near sunspots are called “plage” (French for “beach”).
Sunspot Number May Influence Terrestrial Climate

- More sunspots means *more* light—bright *faculae* ("little torches") outweigh dark sunspots. Rough explanation: toward the limb, strong magnetic fields create a sort of window into the deep, hot sides of convection cells.
- Just one more reason why understanding solar magnetism is important
The Solar Corona
Why is the corona hot?

• Observation: coronal radiation implies very high temperatures
  – Unusual spectral lines can be traced to highly ionized atoms, e.g., Fe XIV
  – The corona is bright in X-rays with an equivalent blackbody temperature ~10^6 K

• Heat cannot just flow to a region of higher temperature
  – Violates the 2nd law of thermodynamics!

• Something must be doing mechanical work on the plasma
  – Magnetic energy is dominant in the corona
  – Work can be done against Lorentz forces to build up magnetic energy further
  – Ohmic heating of the plasma occurs where current is flowing
  – Points to a heating mechanism mediated by magnetic fields

• Two possible scenarios:
  – Waves from the photosphere (and below) travel up along the magnetic field, depositing energy as they go
  – Flares, microflares, nanoflares… solar flares of all scales are always happening, leading to magnetic reconnection and heating
Competing Models of Coronal Heating
Problems with the Heating Models
Due to the low resistivity of the corona

- The corona makes a very good cavity for trapping waves, but not for dissipating them.
  - Magnetosonic waves don’t propagate up through the chromosphere.
  - Shear Alfven waves propagate but are scarcely damped.
- Reconnection rates are slow. Nanoflares are not (yet) observed.

Heating mechanisms are different in these alternatives
- For waves, need damping by anomalous resistivity
- For braiding, need sudden releases via reconnection at current sheets, as in Sweet-Parker model.

"Self-organized criticality"?
Power law distribution is typical.
MHD Magnetic Energy Equation – 1

The starting point is the full electromagnetic energy equation, with no approximations, which can be derived from Maxwell’s equations.

Magnetic energy density = \( B^2 / 8\pi \). Begin with Faraday’s law,
\[
\frac{\partial B}{\partial t} = -\nabla \times E
\]
Take \( B \cdot \) of this
\[
\frac{\partial}{\partial t} \left( \frac{B^2}{8\pi} \right) = -\frac{c}{4\pi} B \cdot \nabla \times E
\]
Use \( \nabla \cdot \frac{A}{\rho} = \frac{\nabla \times B}{\rho} - \nabla \cdot (\nabla \times B) \)
\[
= -\frac{c}{4\pi} E \cdot \nabla \times B + \frac{c}{4\pi} \nabla \cdot (B \times E)
\]
Ampere’s law:
\[
\nabla \times B = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial E}{\partial t}
\]
(Note we’re assuming \( \mathbf{D} = E \) and \( \mathbf{H} = B \) throughout.)
MHD Magnetic Energy Equation – 2

\[
\frac{1}{2} \left( \frac{\mathbf{B}^2}{8\pi} \right) = -\frac{c}{4\pi} \cdot \frac{1}{2} \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} - \mathbf{j} \cdot \mathbf{E} - \frac{c}{4\pi} \nabla \cdot (\mathbf{E} \times \mathbf{B}) \\
\frac{1}{2} \left( \frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi} \right) = -\mathbf{j} \cdot \mathbf{E} - \frac{c}{4\pi} \nabla \cdot (\mathbf{E} \times \mathbf{B})
\]

So far this is \(1\) work + \(2\) \(\nabla \mathbf{E}\) (Poynting flux)

nothing more
than straightforward \(E+M\) theory

Relate to MHD through particular Ohm’s law, plus assumption that \(\partial \mathbf{E}/\partial t\) is negligible.

\(\mathbf{j} = \sigma \mathbf{E}' = \sigma \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right)\)

in fluid frame \(\rightarrow\) in “lab” or fixed frame, via Lorentz transform.
MHD Magnetic Energy Equation – 3

$$\mathcal{E} = \frac{c}{4\pi \sigma} \nabla \times \mathbf{B} - \frac{1}{c} \mathbf{v} \times \mathbf{B} = \frac{\mathbf{j}}{\sigma} - \frac{1}{c} \mathbf{v} \times \mathbf{B}$$

$1. \quad \frac{1}{c} \int_{S} \mathbf{E} = \frac{\mathbf{j}^2}{\sigma} + \frac{1}{c} \mathbf{j} \cdot (\mathbf{v} \times \mathbf{B})$  and

$2. \quad -\frac{c}{4\pi} \nabla \cdot (\mathbf{E} \times \mathbf{B}) = -\frac{c}{4\pi} \nabla \cdot \left( \frac{\mathbf{j} \times \mathbf{B}}{\sigma} \right) + \frac{c}{4\pi} \frac{1}{c} \nabla \cdot [\mathbf{v} \times \mathbf{B} \times \mathbf{B}]$

Meaning becomes clearer when we rewrite

$$\frac{1}{c} \int_{S} (\mathbf{v} \times \mathbf{B}) = -\frac{1}{c} \mathbf{v} \cdot (\mathbf{j} \times \mathbf{B}) = -\mathbf{v} \cdot \mathbf{F}_{\text{Lorentz}}$$

; this term is work done by the fluid against the Lorentz force. (In the equation for kinetic energy, this term appears with a (+) sign; work done by the Lorentz force on the fluid.)
MHD Magnetic Energy Equation – 4

\[
\dot{\frac{1}{2}} \frac{\mathbf{B}^2}{\mu_0} = \text{ohmic heating}, \quad \frac{\partial}{\partial t} \mathbf{B}^2 - \frac{1}{2} \nabla \cdot \left( \mathbf{B} \times \mathbf{B} \right)
\]

Rewrite in terms of \( \nabla \times \mathbf{B} \); let \( \eta = \frac{c^2}{4\pi^2} \)

Note that
\[
\frac{\partial}{\partial t} \left( \frac{\mathbf{B}^2}{\mu_0} \right) = -\frac{\eta}{4\pi} \left| \nabla \times \mathbf{B} \right|^2 - \frac{1}{4\pi} \nabla \cdot \left[ \left( \nabla \times \mathbf{B} \right) \times \mathbf{B} \right]
\]

\[
- \nabla \cdot \left[ \frac{\eta}{4\pi} \left( \nabla \times \mathbf{B} \right) \times \mathbf{B} \right] - \frac{1}{4\pi} \left( \nabla \cdot \mathbf{B} \right) \mathbf{B}
\]

\[
- \nabla \cdot \left[ \frac{1}{4\pi} \mathbf{B}^2 \mathbf{v} \right]
\]

Used identity for \( \mathbf{A} \times (\mathbf{B} \times \mathbf{c}) \)

Combine: \( \frac{\mathbf{B}^2 \mathbf{v}}{4\pi} \)
After combining and rearranging all terms that involve $\mathbf{v}$, the result is:

\[
\left( \frac{1}{\sigma} + \mathbf{v} \cdot \nabla \right) \frac{B^2}{8\pi} = -\frac{J^2}{\sigma} - \frac{B^2}{4\pi} (\nabla \cdot \mathbf{v}) - \frac{c}{4\pi} \nabla \cdot \left( \frac{\mathbf{j} \times \mathbf{B}}{\sigma} \right)
\]

Notice that the original Poynting flux due to $\mathbf{v}$ has been largely cancelled out by terms representing work against the Lorentz force!

Only two terms with $\mathbf{v}$ are left:

- A simple advection term (moved to the left-hand side)
- A term representing loss of magnetic energy due to sideways spreading of flux

The only real energy sink is ohmic heating, $j^2/\sigma$

One can equally well derive this result from the MHD induction law
Alfvén Waves in MHD – 1

Alfvén Waves - Ideal

Assume background field \( B_z = \text{const.} \) in space, time
\[
\frac{1}{4\pi} (\nabla \times B) \times B \approx \frac{1}{4\pi} (\nabla \times B') \times B_z \hat{z} \quad B' = \text{fluc' n.}
\]
\[
= \frac{1}{4\pi} B_z \frac{2}{3} \frac{\partial}{\partial z} B' \quad \text{Coriolis force due to perturbing } B_z.
\]
assume \( |B'| \ll B_z \).

\[
\frac{\partial u'}{\partial t} + u' \nabla \cdot u' = -\frac{1}{\rho_0} \nabla p + \frac{1}{\rho_0} \frac{2}{3} \frac{\partial}{\partial z} B'
\]
\(2\text{nd order in fluc' n.}

assume const. background density, pressure.

neglect \( p \), can get rid of it by taking curl.
Alfvén Waves in MHD – 2

\[ \frac{dB}{dt} = \nabla \times (v \times B) = -v \cdot \nabla B + B \cdot \nabla v \]

Again, \( v = v', B = B_0 + B' \). Only 1st order term.

\[ \frac{dB}{dt} = B_0 \frac{\partial}{\partial t} v' \]

To simplify, let \( B' = B_x \), \( v' = v_x \) due to initial conditions.

\[ \frac{\partial v_x}{\partial t} = \frac{B_0}{\eta \mu_0} \frac{\partial B_x}{\partial z}, \quad \frac{\partial B_x}{\partial t} = B_0 \frac{\partial v_x}{\partial z} \]

\[ \frac{\partial^2 v_x}{\partial t^2} = \frac{B_0}{\eta \mu_0} \frac{\partial}{\partial z} \left( \frac{\partial B_x}{\partial t} \right) = \frac{B_0^2}{\eta \mu_0} \frac{\partial^2 v_x}{\partial z^2} \]

\[ \frac{\partial v_x}{\partial t^2} - \frac{v_x^2}{\eta \mu_0} \frac{\partial^2 v_x}{\partial z^2} = 0, \quad v_x^2 = \frac{B_0^2}{\eta \mu_0} \]

These are transverse waves (as written here).
Alfvén Waves in MHD – 3

Alfvén Waves - Damped

Take above equations and assume dependence $e^{i(\omega t - kz)}$.

Include resistive term in induction eqn.

$$i\omega \hat{V}_x = \frac{B_x}{\eta \pi \rho_0} (-ik) \hat{B}_x, \quad \omega \hat{B}_x = \frac{B_2}{\eta} (-ik) \hat{V}_x - \eta k^2 \hat{B}_x$$

$$\hat{B}_x = -B_2 \frac{k}{\omega} \hat{V}_x + \frac{i\eta k^2}{\omega} \hat{B}_x \quad \text{solve this for } \hat{B}_x$$

$$\hat{V}_x = -B_2 \frac{k}{\omega} \hat{V}_x \left(1 - \frac{i\eta k^2}{\omega}\right)^{-1}$$

$$i\omega \hat{V}_x = \frac{B_2}{\eta \pi \rho_0} (-ik) \left(-B_2 \frac{k}{\omega}\right) \hat{V}_x \left(1 - \frac{i\eta k^2}{\omega}\right)^{-1}$$

$$\omega (\omega - i\eta k^2) = \frac{B_2^2}{\eta \pi \rho_0} (+ik^2) \left[\frac{\omega^2}{k^2} - i\eta \omega = V_A^2\right]$$

$$\omega^2 - i\eta k^2\omega - k^2 V_A^2 = 0 \quad \text{quadratic eqn. for } \omega$$
Alfvén Waves in MHD – 4

\[ \omega = \frac{i\eta k^2 \pm \sqrt{-\eta^2 k^4 + 4|k|^2 v_a^2}}{2} \]

\[ \eta = 0 \Rightarrow \omega = \pm k v_a \sqrt{i} \]

\[ \omega = k v_a \left[ \frac{i\eta k}{2v_a} \pm \sqrt{1 - \frac{\eta^2 k^2}{4v_a^2}} \right] \]

Real part: \( \omega_r = \pm k v_a \sqrt{1 - \frac{\eta^2 k^2}{4v_a^2}} \)

Imag part: \( \omega_i = k v_a \left( \frac{i\eta k}{2v_a} \right) \sim \omega_r (Lu)^{-1} \)

\( Lu \equiv \) Lundquist no. \( = \frac{L v_A}{\eta} \) : very big in polar corona

\( (\frac{i\eta k}{2v_a})^2 \) is a small correction to \( \omega_r \approx \pm k v_a \left[ 1 - \frac{1}{2} \left( \frac{v_i k^2}{2v_a} \right) \right] \)

Note \( e^{i\omega t} = \exp \left( i \left( \frac{\eta k^2}{2} \right) t \right) = \exp \left( -\frac{\eta k^2}{2} t \right) \sim (Lu)^{-1} \omega t! \)

Exponentially damped, as expected.
Why Are the Alfvén Waves Damped?

- Ideal wave (below) depends on transverse, “frozen-in” displacements of $B_z$
- Resistivity weakens the necessary currents, causing the amplitude to slip

\[
\begin{align*}
\text{max} & & \text{max} & & \text{max} & & \text{max} & & \text{max} \\
B_x & & -j_y & & -B_x & & j_y & & B_x \\
\end{align*}
\]

\[\text{max} \quad \text{max} \quad \text{max} \quad \text{max} \quad \text{max} \]

- Field line displacements and velocities

1) Ideal right-traveling wave goes like $\exp(i\omega t - ikz) \rightarrow j_y = -ikB_x (c/4\pi)$

2) Using $\omega = kv_A$ : $i\omega v_x = j_y B_z / (c\rho_0)$, $ikv_A v_x = -kB_x (v_A^2 / B_z)$, $v_x / v_A = -B_x / B_z$

3) Integrate over $dt$ to show that fluid and field line displacements are equal
The Lundquist Number in the Solar Corona

• The Lundquist number is the dimensionless ratio of two timescales:
  – Alfvén wave travel time over a distance $L$
  – Resistive diffusion time over the same distance

• It is equal to the magnetic Reynolds number divided by the Alfvén Mach number, $R_m/M_A$

How large is $L_u$? Need to know resistivity of solar plasma. Spitzer (1962) formula for H plasma gives

$$\eta = 5.2 \times 10^7 \text{ m}^2 / \text{sec}$$

[from Zirin & Stix]

where $\ln \Lambda = 5$ for C2, 10 for chromoph., 20 for corona.

$$L_u = \frac{V_A L}{\eta} = \frac{10^6 \text{ m/sec} \cdot 10^8 \text{ m}}{10^5 (10^6)^{-3/2}} = 10^{14} \text{ m corona}$$
Estimate of Heating Rate Due to Alfvén Wave Damping

L be size of typical coronal loop, while ω ~ (5 min)^{-1}
to get appreciable power from convection.

ωL = \frac{10^5 \text{ km}}{5 \text{ min}} = 300 \text{ km/sec}, about right for V_A
in corona. (Can be around 2000 km/sec in upper corona.)

Power loss / volume due to resistive damping is \frac{\dot{\Omega}}{\Omega}.

\frac{1}{\epsilon} \dot{\Omega} = \frac{1}{\epsilon} \left( \frac{C}{4\pi} \nabla \times B \right)^2 = \frac{1}{\epsilon} \left( \frac{C}{4\pi} \right) k^2 \hat{B}_x^2 = \frac{\eta}{4\pi} \frac{k^2 \hat{B}_x^2}{\epsilon}

Now \eta = \frac{V_A}{L} (L^2 - L^2) and \dot{\Omega} = \frac{1}{\epsilon} \frac{\dot{\Omega}}{\epsilon}

\frac{\dot{\Omega}}{\epsilon} = \frac{V_A}{L} (L^2 - L^2) \hat{B}_x^2 \frac{1}{4\pi}. \quad \text{Comparable to answer}

Exponentially decaying wave is identical: \frac{\partial}{\partial t} \left| B_x^2/(8\pi) \right| = 2\omega \left| B_x^2/(8\pi) \right|

...How does this stack up against the nanoflare/reconnection model?
Sweet-Parker Model of Reconnection – 1

It’s only a 2D model, but it takes into account that the reconnection region must be very thin when the diffusivity is extremely low.

How big can the inflow be, given these geometric constraints?
Sweet-Parker Model of Reconnection – 2

We estimate the layer thickness from MHD magnetic induction, and the outflow speed by assuming it is driven by the Lorentz force.

\[
\frac{V_B}{B} \sim \frac{\eta B}{8} \Rightarrow S \sim \frac{\eta}{V}
\]

\[
V \sim \frac{S}{L} V_A \sim \frac{\eta}{\sqrt{L}} V_A
\]

\[
\Rightarrow V \sim \left( \frac{\eta V_A}{L} \right)^{1/2} = V_A \left( \frac{\eta}{V_A L} \right)^{1/2} = V_A \left( \frac{L_i}{\eta} \right)^{1/2}
\]

where \( L_i \) = Lundquist number \( \equiv \frac{V_A L}{\eta} \).

Finally, need to balance inflow rate with diffusion (reconnection):

Because magnetic pressure drives flow:

\[
\frac{B^2}{8\pi} \sim \frac{1}{2} \rho V^2
\]

\[
\mu \sim \frac{B}{\gamma 4\pi \rho} \equiv V_A
\]

Once again, the Lundquist number comes into play…
Unfortunately, the low rate of magnetic energy conversion is reduced even further if $L$ also approximates distance \textit{between} current sheets:

$$\text{energy} \quad \text{vol. time} = \frac{S}{L} \cdot \frac{1}{L^3},$$

Since $S/L \sim (Lu)^{-1/2}$,

$$\frac{\text{energy}}{\text{vol. time}} = \frac{V_A}{L} \cdot (Lu)^{1/2} \cdot \frac{B^2}{4\pi},$$

It is possible to improve the $(Lu)^{-1/2}$ to $\ln(Lu)$ through better models, such as the ones by Petschek or by Sonnerup and Priest, which have refinements:

- The plasma is compressible—fast or slow magnetosonic shocks allow $u > v_A$
- The incoming magnetic field is bent by shocks, so outflow is broader (in 2D)
Spicules/Fibrils (on the limb/disk)

A possible effect of sound waves on the solar atmosphere

Short-lived, tall jets in the H\(\alpha\) chromosphere may be driven by \(p\)-modes

Credit: Royal Swedish Academy of Sciences
*See QuickTime movie of spicules in action
Filaments and Prominences Viewed in $H_\alpha$
They are condensations of cooler gas suspended in the corona

Source: NOAA/SEL/USAF

HAO A-905
Prominences Can Be Very... Prominent!

4 June 1946: Hα photograph

Source: High Altitude Observatory Archives
HAO A–007
Filaments Tend to Form on Magnetic Neutral Lines
This gives us a clue about what holds them up

Source: NSO and NOAA/SEL/USAF

Steve Lantz
Electrical and Computer Engineering 5860
www.cac.cornell.edu/~slantz
Huge Eruptive Prominence Captured by STEREO

*QuickTime movie shows all the action*
Eruptive Prominence from SDO First Light

Zoomed-In Animation of Eruptive Prominence
Watch for the twist in the plumes of plasma as they descend

*QuickTime movie shows the event
A Coronal Mass Ejection Witnessed by SOHO/LASCO

CME events are often associated with eruptive prominences.
Coronal Structures – 1
Possible MHD equilibria for long-lived formations

Magnetic fields dominate \( \Rightarrow \) equilibrium must have no influx force, \( \vec{j} \times \vec{B} = 0 \).

If \( \vec{j} = 0 \): Potential field, lowest energy state. (to relax to it)

\[ \nabla \times \vec{j} = 0, \quad \nabla \times \nabla \times \vec{B} = 0, \quad -\nabla^2 \vec{B} = 0 \] Laplace eqn.

If \( \vec{j} \parallel \vec{B} \): "force-free" equilibrium

\[ \nabla \times \vec{B} = \alpha \vec{B} \] can have \( \alpha = \text{const.} \) or \( \alpha = \alpha(x) \)

Note \( \alpha = 0 \Rightarrow \) potential field.

Both types of configurations are important for modeling the corona. Typical problem: given photospheric \( \vec{B} \), construct 2D magnetic fields in corona.
Coronal Structures – 2
Prominences and their eruption

Can get the prominence to eject by *squeezing* the footpoints
Coronal Structures – 3
Creating a solar flare

Can get the arcade to flare by shearing or twisting the footpoints

Coronal structures and dynamics can have consequences for Earth…

• Equilibrium structures ( prominences, arcades) can suddenly lose stability, ejecting plasma and/or radiation into interplanetary space

• Low-level disturbances (waves, nanoflares) apparently heat the steady-state corona to high temperatures
  – This turns the corona into a much stronger X-ray source than the photosphere
  – As we will see, it drives a steady-state plasma outflow, the solar wind
Solar Wind Formation

First look at hydrostatic, extended corona, \( \frac{v}{c} = 0 \)
and pressure force is balanced by gravity:

\[-\frac{\partial p}{\partial r} - p \left( \frac{G M_0}{r^2} \right) = 0 \quad \text{assume} \quad p = \frac{\rho}{\bar{m}} \]

\[\frac{1}{\rho} \frac{\partial p}{\partial r} = - \frac{G M_0 \bar{m}_p}{2kT} \cdot \frac{1}{r^2} \]

\[\ln p = \frac{G M_0 \bar{m}_p}{2kT} \frac{1}{r} + K \]

\[\ln \frac{p}{p_0} = \frac{G M_0 \bar{m}_p}{2kT} \left( \frac{1}{r} - \frac{1}{R} \right) \]

\[p(r) = p_0 \exp \left\{ \frac{G M_0 \bar{m}_p}{2kT} \left( \frac{1}{r} - \frac{1}{R} \right) \right\} \]

problem: as \( r \to \infty \), \( p \to p_0 \exp \left\{ - \frac{G M_0 \bar{m}_p}{2kTR} \right\} \)

for coronal \( T \) of \( 10^6 \), this is \( \approx p_0 e^{-8} \approx 3 \times 10^{-4} p_0 \) far higher than \( p \) of ISM: mismatch.
Parker (1958) Solar Wind Equation – 1

Assume flow is steady, isothermal, depends on $r$ only

\[ \frac{\partial}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho \mathbf{v} \right) = 0 \]

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\partial \mathbf{p}}{\partial r} - \frac{\rho GM_0}{r^2} \quad \text{neglecting} \ B. \]

Also need eqn. of state, $\rho = \rho \frac{RT}{\mu}$

$\mu =$ avg. molec. weight in amu, now: $\mu \approx 0.5 \ \text{amu}$

From first eqn.,

\[ \frac{\partial}{\partial r} r^2 \mathbf{v} = -\rho \frac{\partial}{\partial r} (r^2 \mathbf{v}) \]

or can integrate,

\[ 4\pi r^2 \rho v = \text{const} \quad \Rightarrow \]

from eqn. of state

\[ \frac{dp}{\partial r} = \frac{RT}{\mu} \frac{dp}{dr} = \frac{RT}{\mu} \left( -\frac{\rho}{r^2} \frac{\partial}{\partial r} (r^2 \mathbf{v}) \right) \]
Parker (1958) Solar Wind Equation – 2

\[
v \frac{dv}{dr} = \frac{RT}{\mu} \frac{1}{r^2} v \frac{d}{dr} (r^2 v) - \frac{GM_0}{r^2}
\]

\[
= \frac{RT}{\mu} \frac{2\pi v}{r^2} + \frac{RT}{\mu} \frac{1}{v} \frac{dv}{dr} - \frac{GM_0}{r^2}
\]

\[
(1 - \frac{RT}{\mu v^2}) v \frac{dv}{dr} = \frac{2RT}{\mu} \frac{1}{r} - \frac{GM_0}{r^2}
\]

Define \( c_s^2 = \frac{RT}{\mu} \), like a sound speed. \( c_s^2 = \frac{gRT}{\mu} \)

Goal: solutions for which \( \rho = \frac{\text{const.}}{4\pi r^2 v} \rightarrow 0 \), \( \rho \rightarrow 0 \)

for large \( r \). This says \( v \sim r^g \) where \( g > -2 \)

But also: need transonic flow solution (sub \( \rightarrow \) super)

like flow in a Naval nozzle - diverging geometry

High \( \frac{v}{c_s} \rightarrow v < c_s \) \( \lim_{r \rightarrow 0} \frac{\rho}{\mu} \frac{T}{v} \)
The subsonic “solar breeze” solution is also permitted but is not observed
Spiral Magnetic Field in the Solar Wind – 1

Consider a non-rotating Sun + Solar Wind. Magnetic field lines would be helical, radially outwards until straight—only steady state possible (field lines join at \( \infty \)).

Rotating Sun: still have \( \mathbf{u} \parallel \mathbf{B} \) in rotating frame of reference (field lines are anchored to rotating Sun). If \( \mathbf{u} \) isn't \( \parallel \mathbf{B} \) \( \Rightarrow \) \( \mathbf{B} \) varies in time.

Assuming \( \mathbf{\mu} \parallel \mathbf{B} \) is fixed frame, then in rotating frame, \( \mathbf{\mu}_r = -\omega \mathbf{r} \). (Note that this is a weird assumption. Normally, if you pitched something off a merry-go-round, e.g., it would have \( \mathbf{\mu}_r = +\omega \mathbf{r} \), \( \mathbf{\mu}_r = 0 \). Here things are constructed so that the angular momentum of the solar wind is zero.) (You push it off backward, so you speed up the Sun by some tiny amount!)
Spiral Magnetic Field in the Solar Wind – 2

Let’s try several assumptions:

1. $u_{\phi} = -\omega r$. Easiest to compute at $r=R$.
2. $u_{\phi}(r) = 0$, i.e., velocity component in fixed frame is $+\omega R$, instantaneous vel. at radius $r$ where solar wind parcel leaves Sun tangential.

(1) Show that field lines are Archimedean spirals—the same pattern made by streams of water from a rotating lawn sprinkler when viewed from above:

\[ \frac{u_{\phi}}{u_r} = \frac{B_{\phi}}{B_r} \] (have dropped the primes). Let $u_r = \text{const.}$

\[ -\frac{\omega r}{u_r} = \frac{B_{\phi}}{B_r} \] Eqn. of field line:

\[ \frac{r d\phi}{B_{\phi}} = \frac{dr}{B_r} \]

\[ \frac{r d\phi}{u_r} = -\frac{\omega r}{u_r} \]

\[ d\phi = -\frac{\omega r}{u_r} \, dr \]

\[ \Phi - \Phi_0 = -\frac{\omega r (r - R)}{u_r} \]

where $\Phi$ is longitude of field line leave Sun of corona at $R$.
The equation \( r = R - \frac{u}{\omega} (\Phi - \Phi_o) \) is the eqn. of an Archimedean spiral. Yet another result by Parker!

(2) Modify (1) so that \( \Phi \) in fixed frame \( \neq 0 \), but \( = \frac{\omega R^2}{r} \)

\[
\frac{d\Phi}{dr} = \frac{\omega (R^2 - r)}{u}, \quad d\Phi = \frac{\omega R^2}{u} \frac{dr}{r^2} - \frac{\omega}{u} \frac{dr}{r}
\]

\[
\Phi = -\frac{\omega R^2}{u} \ln r - \frac{\omega}{u} R + K''
\]

Let \( \Phi = \Phi_o \) at \( r = R \)

\[
\Phi_o = -\frac{\omega R^2}{u} \ln R - \frac{\omega}{u} R + K''
\]

Subtract eqns.

\[
\Phi - \Phi_o = \frac{\omega R}{u} (1 - \frac{R}{r}) - \frac{\omega}{u} (r - R)
\]

\[
\Phi - \Phi_o = -\frac{\omega}{u} \left( r + \frac{R^2}{r} - 2R \right)
\]

Sum of an Archimedean spiral and a hyperbolic spiral
Spiral Magnetic Field in the Solar Wind – 4

Magnetic field is advected radially outwards — but Sun is rotating — combines to give

Spiral field lines ↓

"Corotating streams"

dipole field: lines oppositely directed in each hemisphere ⇒ current sheet *

3-D:

"Hat with floppy brim"
The 3D Current Sheet: “Ballerina Skirt”
Ulysses Main Results

- There are two distinct plasma regimes in the solar wind
  - Near the equator, speed (red line) is low and density (blue line) is high. Composition is typical of the corona.
  - At high to mid latitudes, speed is high and density is low, with less variability in both. Composition is typical of the photosphere.
  - Speed is approximately 750 km/s everywhere except near the equator.
- The solar wind’s magnetic field is not based on a dipole
  - A dipole field would be twice as strong over the poles; in the solar wind, it is near-uniform with latitude.

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Spiral Magnetic Field in the Solar Wind – 5

At 1 AU:

- Open/fast region (extended helios)
- 2 "sector crossings" per solar rotation (inward/outward B)
- Slow/closed region (extended streamers)

Fast streams create compressions in B, p, leading to shock fronts in distant solar wind. Quick rise, gradual fall-off (to rarefaction).
Shocks in the Interplanetary Medium

Where a fast corotating stream follows a slow one
• Sector crossings involving shock compression have a greater effect on geospace than those involving rarefactions
• To see why, first need to understand the steady state of interactions between the near-Earth environment and the solar wind…
Chapman-Ferraro (1930) Magnetosphere
(Figure from Chapman and Bartels, 1940)

- Solar wind is like a superconductor that excludes the sunward dipole field
- At planar boundary, a current sheet forms; field is summed with image dipole
- Separatrix QQ defines the “cusp” latitude associated with auroral ovals
Anatomy of the Earth’s Magnetosphere
Current Sheets at the Magnetopause and Across the Tail

At the “nose” of the magnetosphere, the magnetic pressure of the Earth’s squeezed dipole field can stand up to the ram pressure of the solar wind

Figure from Kivelson and Russell
Coronal Mass Ejections
How to launch a “magnetic cloud”

Open field lines \(\text{vs.}\) closed field lines

- "coronal holes"
- "loops"

Steady solar wind

Can pinch off into "coronal mass ejections"

Big disruption when they hit the Earth's magnetosphere

Preceded by shock

Polar cap wind

If aimed at Earth, a CME drastically changes the momentum (velocity and density) of the solar wind that impinges on the magnetosphere
Magnetic Reconnection and Plasmoid Ejection

Magnetic could contains *southward* IMF

*Play QuickTime movie of solar wind gusts hitting the magnetosphere*
Cusp Aurora Due to Reconnection at High Latitude

Magnetic cloud contains *northward* IMF

http://web.ift.uib.no/~nikost/research.html
Sun-Earth System Is Driven by the 11-Year Solar Cycle

**Solar Maximum:**
- Increased flares, solar mass ejections, radiation belt enhancements.
- 100 Times Brighter X-ray Emissions
  0.1% Brighter in Visible
- Increased heating of Earth’s upper atmosphere; solar event induced ionospheric effects.

**Declining Phase, Solar Minimum:**
- High speed solar wind streams, solar mass ejections cause geomagnetic storms.
First-Ever 3D Images of the Sun from STEREO – NASA's Solar TErrestrial RElations Observatory satellites

STEREO Images – 2
Spicules, Polar Coronal Hole, Prominence
STEREO Images – 3
Active Regions

*See QuickTime movies for 3D animations